RELIABILITY OF TELECOMMUNICATION SYSTEM BY VARIOUS REPAIR REGIME

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Summary In the past the reliability of the telecommunication system was evaluated on the simple assumption of the independence of elements. Computation of steady – state series – system availability depends on specific assumptions made about the non failed items during the system failure. In the article is more exact method of calcul global reliability with consideration of various regimes of maintenance.

1. INTRODUCTION

In telecommunication systems (namely in telecommunication network) very often occur the calcul of reliability series elements evaluated by availability. Steady/state availability (also called: limiting availability) is defined as the long-term fraction of time that an item is available.

When a series systems fails due to the failure of any one of its components, all the other components take a rest and are therefore not at risk of failure.

In the text will be used the following notation:

s	statistical (ly)
n	number of components in the series – system
λ_i	failure rate of component i
μ_i	repair rate of component i
a_i	$\mu_i/(\lambda_i+\mu_i)$, $i=1, 2, \dots n$: steady – state availability of component i
A_{sys}	system steady – state availability
Φ	system matrix of transition rates between states
S_o	system operating state
S_i	system state, item i is failed; $i>0$
p_i	steady – state probabilities of state i
$\boldsymbol{P}$	vector of p_i

2. ASSUMPTIONS

1) Failure of any item in the series constitutes failure of the system (definition of series in reliability terms).
2) At system failure as defined in #1, all other items in the series stop, and cease to be at risk of failure until the failed item is restored and system is re-started.
3) Steady – state availability of an item is the ratio of running time, (same for all items) to: "running time" plus "failure – repair time", (different for each item), over a long, theoretically infinite, period. System free time does not enter the calculations.
4) The maintenance policy for all items is fixed for the period of data collection upon which the estimates of item steady – state availability are based. The policy can be failure – only.
5) Items in this context can be complex assemblies or simple parts. The theoretical basis changes but the practical results does not).

3. MODEL DESCRIPTION

Consider a series – system of n s-independent components, i. e, a failure of any component causes a system failure [1], [2].

Case – 1: Non-failed components with an operational state during repair.

Components are repaired immediately at failure while other non-failed components continue to operate, or at least remain energised, and can fail (and hence, age) during the repair of the original failed components. The steady – state availability for a series – system of n s-independent components is the product of the component availabilities.

$$A_{sys} = \prod_{i=1}^{n} a_i = \prod_{i=1}^{n} \frac{\mu_i}{\lambda_i + \mu_i} \quad . \tag{1}$$

Case – 2: Non-failed components with a switch-off during repair.

Components are repaired immediately at failure while other non-failed components are also immediately taken out of operation, or at least switched-off and cannot fail (therefore do not age) until the failed component (s) is repaired.

A series system with constant λ_i, μ_i, can be represented by a matrix of conditional rates of change of state from S_i (row) to S_j (column), as

$$\Phi \equiv \begin{bmatrix} -\sum\lambda & \lambda_1 & \lambda_2 & \dots & \lambda_{n-1} & \lambda_n \\ \mu_1 & -\mu_1 & 0 & \dots & 0 & 0 \\ \mu_2 & 0 & -\mu_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu_{n-1} & 0 & 0 & \dots & -\mu_{n-1} & 0 \\ \mu_n & 0 & 0 & \dots & 0 & -\mu_n \end{bmatrix} \tag{2}$$

The p_i, $i=0, 1, \ldots n$ for a system represented by a matrix such as (2), having no absorbing state can be found from:

$$P\,x\,\Phi=0, \qquad \sum p_i = 1 \;. \tag{3}$$

In this article, $\lambda_i \cdot p_0 = \mu_i \cdot p_i$, for all $i=1, 2, \ldots n$, and by solving the simultaneous equations which are the expansion of (3):

$$\frac{1}{p_0} = \frac{1}{A_{sys}} = 1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i};$$
$$\frac{\lambda_i}{\lambda_i+\mu_i} = 1 - a_i; \frac{\mu_i}{\lambda_i+\mu_i} = a_i; \qquad i=1, 2, \ldots n \tag{4}$$

we obtain

$$A_{sys} = \left[1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\right]^{-1} = \left[1 + \sum \frac{1-a_i}{a_i}\right]^{-1} \;. \tag{5}$$

For example for two elements is:

$$Px\Phi = [p_0 \quad p_1 \quad p_2] x \begin{bmatrix} -\lambda_1 - \lambda_2 & \lambda_1 & \lambda_2 \\ \mu_1 & -\mu_1 & 0 \\ \mu_2 & 0 & -\mu_2 \end{bmatrix} =$$
$$= 0$$

$$-p_0(\lambda_1+\lambda_2) + p_1\mu_1 + p_2\mu_2 = 0$$
$$p_0\lambda_1 - p_1\mu_1 = 0$$
$$p_0\lambda_2 - p_2\mu_2 = 0$$

and $\quad p_0 + p_1 + p_2 = 1$,

$$\frac{1}{p_0} = A_{sys} = \left[1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right]^{-1}.$$

It can be proved that for $n=1$ A_{sys} after equation (1) is equal after equation (5) and for $n>1$ A_{sys} after equation (1) is less as after equation (5).

4. CONCLUSION

It was discovered the importance of the difference between the "product rule" after equation (1) and the "correct series availability" after equation (5).
Compare the result for 400 items in series all of steady – state availability 0,999. For such a system that operates continuously if available, the best way to perceive the practical difference is as: 350 hours/year of system operation, which the product - rule system would be downtime. The items in a practical system would not have availabilities all of the same order of magnitude. The effect of even a few items with availabilities of 0,99 rather than 0,999 renders the difference unimportant, but those are the ones that should attract redundancy.
In the next work will be interessant to take in consideration economical cost of items.

REFERENCES

[1] L. Kleinrock: Theory in Queuing Systems: John Wiley & Sons, 1975, Vol I. p. 59

[2] Hoang Pham: Commentary: Steady - State Series – System Availability. IEEE Transaction on Reliability, Vol. 52, No 2, June 2003, pp. 146 – 147