PROPAGATION OF MICROWAVES THROUGH ATMOSPHERIC ENVIRONMENT

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Summary Wireless microwave systems tend to have high availability figures, but at the expense of the ability to operate at higher data rates. A quality of free space communication depends on atmospheric conditions. It is shown that microwave attenuation reacts sensitively on changes of hydrometeor size distribution. However, a signal transmission is also significantly affected by both, refractive index and shape of hydrometeors. Non-spherical particles attenuate radiation at frequencies higher than about 200 GHz more effectively than volume equivalent spheres. On the other hand, the non-spherical ice phase hydrometeors form weakly attenuating media at $\nu < 150$ GHz. The attenuation coefficient for small non-spherical particles is approximately two times smaller than the corresponding coefficient for spheres. The quality of microwave signal transmission through icy cloud is therefore much better as when the communication path is realized through rainy atmosphere (the particle sizes are assumed to be the same in both cases).

Keywords: free space communication, microwave scattering and absorption, hydrometeors

1. INTRODUCTION

Microwave links are used as the major trunk channel for long distance communication because of major advantages over cabling systems: i) freedom from land acquisition rights, and ii) ease of communication over difficult terrain. On the other hand, the use of microwave links has a number of disadvantages that mainly rise from the use of free-space communication. The modern urban environment presents a particular challenge, in that bandwidth allocation, radio frequency interference, link obstruction and atmospheric pollution place maximum constraints on the system simultaneously. As for atmospheric effects, the use of free-space communication results in susceptibility to weather effects, particularly rain. Many terrestrial and satellite communication systems operate at microwave frequencies and are considerably affected by hydrometeors in their paths. A medium filled with hydrometeors will attenuate the signal, cause interference between different links because of bistatic scattering, and in frequency reuse systems using transmission on orthogonal polarizations cause crosstalk as a result of depolarization [1]. The paper deals mainly with an influence of hydrometeor shape and size distribution on propagation of microwaves (10-90 GHz) in the atmospheric environment. A successful numerical scheme and the physical model were developed to simulate signal damage in atmospheric environment.

2. ATTENUATION OF MICROWAVES BY HYDROMETEORS

The scattering of electromagnetic waves by hydrometeors (e.g. precipitation particles, raindrops, small-hail particles, sleet, hailstones, and graupel) and their propagation through precipitation media are of fundamental importance in understanding the signal damage. Considerable attention of world-wide research in this area is focused on simulation the hydrometeors more accurately to improve the theoretical models of interaction of electromagnetic radiation with hydrometeors. Their shape, size, fall behavior, and composition must be represented realistically with enough detail that capture the dominant features influencing the measurable signal parameters at the frequencies of interest [2]. Based on the determination of hydrometeor physical and structural characteristics, we can objectively evaluate and describe the process of transfer, scattering and absorption of microwave radiation in the Earth's atmosphere. A presence of hydrometeors significantly influences a quality of communication line in cloudy zone of lower troposphere. The microwave radiation penetrating a cloud is determined by the temperature, size distribution, phase and shape of the hydrometeors. Up to now most microwave radiative transfer models apply Lorenz–Mie theory to calculate the effect of scattering and therefore assume spherical geometry for all hydrometeors. This is a poor approximation when focussing on falling ice phase hydrometeors which influence the microwave signal of the atmosphere strongly. Natural raindrops are excellent examples of homogeneous, but partly irregular particles. These drop shapes can be described by a series of Chebyshev polynoms [3]. However, it is known that small raindrops are nearly spherical.

The attenuation of electromagnetic radiation in the Earth's atmosphere can be simply described as intensity reduction due to extinction. A measured intensity of monochromatic radiation can then be defined as follows [4]

$$I(\lambda)=I_0(\lambda)e^{-\tau(\lambda)M} \quad , \qquad (1)$$

where $I(\lambda)$ is the flux density of radiation detected by receiver (in case of direct transmitter-receiver link), $I_0(\lambda)$ is the incident radiation flux density generated by transmitter, $\tau(\lambda)$ is the atmospheric spectral optical thickness at a given wavelength λ, and M is so-called optical air mass. The function M can be easily computed for known communication path – because it depends mostly on position of both, receiver and transmitter [5,6].

The hydrometeor optical thickness $\tau_H(\lambda)$ is a most unstable part of optical thickness $\tau(\lambda)$ and can be simply calculated as follows

$$\tau_H(\lambda)=\tau(\lambda)-\tau_R(\lambda)-\tau_A(\lambda) \quad , \qquad (2)$$

where Rayleigh component of the total optical thickness is denoted by $\tau_R(\lambda)$, and $\tau_A(\lambda)$ represents the optical thickness produced by absorption effect in atmospheric gases. The optical thickness (called also extinction, or attenuation) is a product of both, absorption and scattering of electromagnetic waves. Generally, the atmosphere not only attenuates the radiation (by the influence of scattering and absorption), but also radiates itself (we are simply speaking about microwave brightness temperature [7]). From the mathematical point of view, the extinction of electromagnetic waves by hydrometeor particles is most complicated effect so we will concentrate especially on computation of $\tau_H(\lambda)$.

3. SPHERICAL HYDROMETEORS

In general, $\tau_H(\lambda)$ is a complex function of particle characteristics, such as size distribution, shape, and chemical composition. Nevertheless, an ensemble of spherical, homogeneous hydrometeors with the same volume distribution often supplies the particle populations of different characteristics. Such approximation can be accepted if the condition $r \ll \lambda$ is satisfied for particle effective radius r. Optical thickness is then a simple function of particle radius r and of incident wavelength λ

$$\tau_H(\lambda)=\pi\int_0^\infty Q_{ext}(r,\lambda,m)r^2 f(r)dr \quad , \qquad (3)$$

where $Q_{ext}(r,\lambda,m)$ is the efficiency factor for extinction, and m is the particle refractive index, which depends on wavelength λ. The fast calculation of efficiency factor $Q_{ext}(r,\lambda,m)$ for spherical particles is based on rigorous Mie's theory [8]. The extinction efficiency for these particles is computed as follows

$$Q_{ext}=\frac{2}{x^2}\Re\left\{\sum_{n=1}^{\infty}(2n+1)(a_n+b_n)\right\} \quad , \qquad (4)$$

where $\Re$ denotes real part of the complex function (inside combined brackets), x is the well-known size parameter ($x=2\pi r/\lambda$) and the coefficients a_n and b_n are computed using Riccati-Bessel functions [9]

$$a_n=\frac{\left[\frac{D_n(mx)}{m}+\frac{n}{x}\right]\psi_n(x)-\psi_{n-1}(x)}{\left[\frac{D_n(mx)}{m}+\frac{n}{x}\right]\xi_n(x)-\xi_{n-1}(x)} \qquad (5a)$$

$$b_n=\frac{\left[m\times D_n(mx)+\frac{n}{x}\right]\psi_n(x)-\psi_{n-1}(x)}{\left[m\times D_n(mx)+\frac{n}{x}\right]\xi_n(x)-\xi_{n-1}(x)} \qquad (5b)$$

with the recurence relations

$$D_{n-1}(mx)=\frac{n}{mx}-\frac{1}{D_n(mx)+\frac{n}{mx}} \qquad (6a)$$

$$\psi_n(x)=\frac{(2n-1)\psi_{n-1}(x)}{x}-\psi_{n-2}(x) \qquad (6b)$$

$$\xi_n(x)=\frac{(2n-1)\xi_{n-1}(x)}{x}-\xi_{n-2}(x) \qquad (6c)$$

and the initial conditions

$$\psi_{-1}(x)=\cos(x) \qquad \psi_0(x)=\sin(x)$$
$$\xi_{-1}(x)=-\sin(x) \qquad \xi_0(x)=\cos(x)$$
$$D_\infty(mx)=0 \quad . \qquad (7)$$

The series are terminated after $x+4\sqrt[3]{x}+2$ terms.

3.1. A role of magnetic permeability

At microwave frequencies the hydrometeor constituents can respond to both electric and magnetic fields, and both the dielectric constant (permitivity) and magnetic permeability can play a role in the attenuation of electromagnetic radiation in case $r \ll \lambda$.

All laboratory measurements on particles at optical and NIR wavelengths are consistent with a relative permability μ of 1.0 in this frequency range. This agrees with theoretical considerations on magnetism [10]. The only relevant permeability in the given wavelength range is probably induced permeability as discussed by Ossenkopf [11] (there exists an erratum for that paper published on May, 1993). In inhomogeneous media electrically induced resonances lead to effective permeability different from 1. Hence, permeability at these wavelengths is rather an effect of geometry than of material. A permeability $\mu > 1$ in the case of electromagnetic waves requires that the change in the H field can produce a corresponding imbalanced dipole moment P. One can write this as

$$\mu \neq 1 \quad \text{if} \quad \ell^2 << \frac{\chi^2 c^2}{\omega^2} \quad , \qquad (8)$$

where ℓ is the size of the particle (or so-called Weiss zone), χ is the magnetic susceptibility, c is the light velocity in the vacuum, and ω is an angular frequency. For diamagnetic materials $\chi \approx v^2/c^2$ (v is the velocity of electrons) – i.e. $\chi^2 = \alpha^2$, where α is Sommerfeld constant. One can easily see that at 1 GHz, the permeability plays a role for micro-sized constituents of hydrometeors. The treatment of a relative permeability of the materials increases the size parameter x of the particles by $\Re\{\sqrt{\mu_{env}}\}$, where μ_{env} is the environment permeability (usually equals to 1). Additionally it increases the refractive index by $\Re\{\sqrt{\mu}\}$, and it changes coefficients a_n and b_n (Eqs. 5a,b) as follows

$$a_n = \frac{\left[\frac{\mu D_n(mx)}{m} + \frac{n}{x}\right]\psi_n(x) - \psi_{n-1}(x)}{\left[\frac{\mu D_n(mx)}{m} + \frac{n}{x}\right]\xi_n(x) - \xi_{n-1}(x)} \qquad (9a)$$

$$b_n = \frac{\left[\frac{m \times D_n(mx)}{\mu} + \frac{n}{x}\right]\psi_n(x) - \psi_{n-1}(x)}{\left[\frac{m \times D_n(mx)}{\mu} + \frac{n}{x}\right]\xi_n(x) - \xi_{n-1}(x)} . \qquad (9b)$$

The series are terminated after $\max(x_{stop}, y_{mod})$ terms, where $x_{stop} = x + 4\sqrt[3]{x} + 2$ and $y_{mod} = \lfloor x.\mu.m \rfloor$. One can see that the number of terms necessary to compute coefficients a_n, and b_n growths up with increase of μ (because of y_{mod}). However the effect is important mainly at wavelengths comparable to the size of particles. Our numerical simulations shown that the position x_{max} of the main (also the first) mode of the efficiency factor for extinction Q_{ext} depends on μ as follows

$$x_{max}(m, \mu) = \frac{x_{max}(m_0, \mu_0)}{m\mu - \frac{1}{m\mu}} \quad , \qquad (10)$$

where the constants m_0 and μ_0 must satisfy condition

$$m_0\mu_0 - \frac{1}{m_0\mu_0} = 1 \quad , \qquad (11)$$

i.e. $m_0\mu_0 \approx 1.62$.

If an appreciable fraction of the particle material is magnetic, then magnetic dipole effects must dominate the efficiency factor for extinction at the frequencies less than approximately 30 GHz, where plausible materials have a magnetic response.

4. NON-SPHERICAL HYDROMETEORS

Raindrops can oscillate and deviate from their equilibrium shape. Oscillating drops can be modeled, on average, as oblate spheroids (aspect ratio = axis ratio < 1) with larger axis ratios compared to the equilibrium shape model [12]. The axial symmetry of spheroidal particles has a direct impact on scattering pattern, which characterizes optical properties of these particles. Spherical particles scatter radiation indepentently on their orientation in the space. Ice phase hydrometeors such as hailstones, graupel particles, and snowflakes may have definitely irregular shapes. The characteristics of radiation scattered by such particles can be calculated using the Discrete Dipole Approximation (DDA) [13]. The most important advantage of this technique is its applicability to arbitrarily shaped, inhomogeneous, and anisotropic particles. However, there are several disadvantages: limited numerical accuracy for certain particle sizes, but especially when calculating the scattering matrix elements; slow convergence of results with increasing the number of dipoles; and the need to repeat the entire calculation for each new direction of incidence. Nevertheless it was shown by Draine and Flatau [14] that, for dielectric materials ($|m| \leq 2$), the DDA permits calculations of scattering and absorption that are accurate to within a few percent. Other effective method, so-called T matrix method [15,16] is almost exclusively applicable to rotationaly symmetric particles. Such method was successfully used to calculate the radiation scattering by general star-shaped particles [17].

We used DDA method to perform microwave scattering computations for ice-phase hydrometeors of strictly non-spherical shape. In general, a precision of the scattering calculations depends on parameter $|m|kd$, where d is an interdipole separation and k is well-known wave number ($k=2\pi/\lambda$). To fulfil the sufficient accuracy of scattering/absorption efficiency factor calculations, the given parameter should be less than 1. Therefore the number of dipoles, which constitute the particle body, must be enlarged when $|m|kd$ reaches this critical level. Unfortunately, the calculation procedure is in such a case considerably time consuming.

5. NUMERICAL RESULTS AND DISCUSSION

Majority of microwave scattering theories for non-spherical hydrometeors deals with model of randomly oriented particles. The most general size distribution commonly employed in microwave precipitation studies is the modified gamma distribution, written as [18]

$$f(r) = Ar^a \exp\{-br\} \qquad (15)$$

Raindrops have equivalent-volume spherical radius ranging from about 0.05 to 5 mm. The amplitude matrix requires specification of a particle complex refractive index $m = m_r + i\, m_i$, where the real part m_r is related to how fast microwave propagates through the species and the imaginary part m_i is related how much radiation the species absorbs. For the refractive index of ice in microwave, m_r has an approximately constant value of 1.78 at all frequencies and temperatures, whereas m_i is weakly frequency and temperature dependent and ranges from about 10^{-4} to 10^{-2} (very weak absorption). The refractive index for liquid water in the microwave has a stronger frequency and temperature dependency, with m_r ranging from between about 3 and 10 and m_i ranging from approximately 0.5 to 3 (strong absorption) for microwave frequencies and common atmospheric temperatures.

The numerical simulations were realized in frequency range from 10 up to 100 GHz, as there is a need to give guidance to engineers in the design of Earth-space telecommunication systems for frequencies higher than 10 GHz. The government strictly regulates the use of the microwave spectrum for communication purposes. Parts of the spectrum between roughly 1 GHz and 50 GHz sections are available for commercial applications and the government grants the right to use a specific microwave band with a limited channel bandwidth through a licensing process.

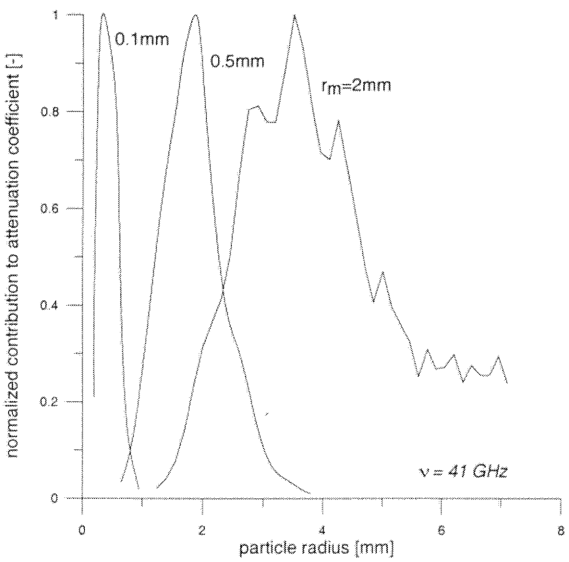

Fig. 1. The contribution of various particle sizes (ice hydrometeors) to total attenuation coefficient at ν=41 GHz (λ=7.2 mm). Underintegral function in Eq. (3) was calculated for three model size distributions according to Eq. (15): a=2 and b=1, 4, 20 [mm^{-1}] for which r_m=2, 0.5 , 0.1 [mm] respectively.

Presented calculations correspond to ν=41 GHz (λ=7.2 mm) - it lies also in the frequency range at which the precipitation radars are operating. An interaction of radiation with spherical hydrometeors was computed using the module Direct1.0 [19]. The module can be also utilized to compute attenuation coefficient at discrete values of $f(r)$ – LaGrange polynomials are simply used to interpolate the distribution function. Here, one must be aware of possible problems in case of small lattice density with r (especially when calculate $\tau_H(\lambda)$ at large wavelengths: $\lambda \approx 1000r$). However, the computation is then realized in Rayleigh-zone because of $r \ll \lambda$, where Q_{ext} is simply proportional to r^4/λ^4. One can easily find the expression $\tau_H(\lambda) = (\pi/\lambda^4) \int r^6 f(r)\, dr$, if accept $r \ll \lambda$. The contribution of hydrometeors to $\tau_H(\lambda)$ will be still significant until $f(r)$ decreases slower than r^{-8}. The cut off point for quantity $r^6 f(r)$ is defined well for gamma function because of rapid (exponential) reduction of particle concentration at large values of r.

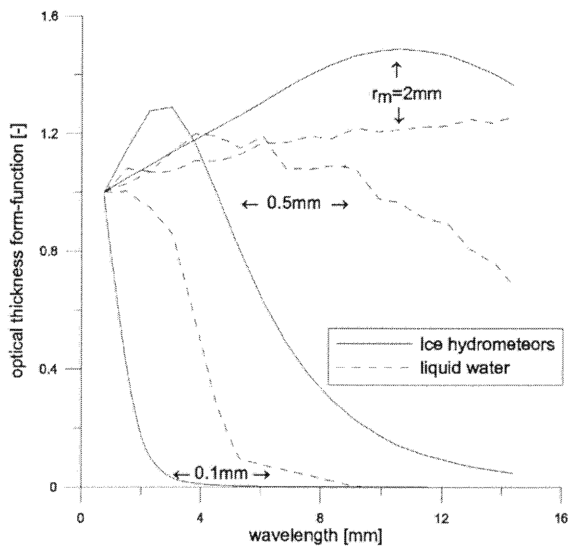

Fig. 2. The dependency of optical thickness form-function on signal wavelength (ν decreases up to 20 GHz) in rainy atmosphere. Computation was realized for three model size distributions according to Eq. (15): a=2 and b=1, 4, 20 [mm^{-1}] for which r_m=2, 0.5 , 0.1 [mm] respectively.

Particles of different size contribute to the radiation attenuation in different ways. In general, particles of size close to the modal radius r_m have a dominant contribution in the total attenuation. The contribution of various particle sizes to attenuation coefficient is drawn in Fig. 1, where three gamma functions are taken into account (r_m = 0.1, 0.5, and 2 mm). The contribution-function is very narrow if r_m is more than one order in magnitude smaller than wavelength of transmitted signal. A most dominant contribution of under-integral function to total attenuation coefficient (Eq. 3) can be observed at particle radii

$$r \approx \frac{\pi^2}{8} \frac{\sqrt{1+10r_m}-1}{m\mu - \frac{1}{m\mu}} \quad \text{if} \quad r_m \leq \frac{\lambda}{10} \quad . \qquad (16)$$

This approximation is applicable also to $r_m \approx \lambda$ in certain degree of accuracy.

Calculation of dimensionless optical thickness form-function $\tau^F{}_H$ is presented in Fig. 2 for the three gamma distributions. The curves represent a behavior of the optical thickness, which can be observed in rainy atmosphere. Real attenuation coefficient (measured in db/km) is proportional to $\tau^F{}_H$ at a given wavelength, so one can simply write $\tau_H = K\tau^F{}_H$, where proportionality quotient K depends on rainfall (eventually snowfall) rate nearly in linear way.

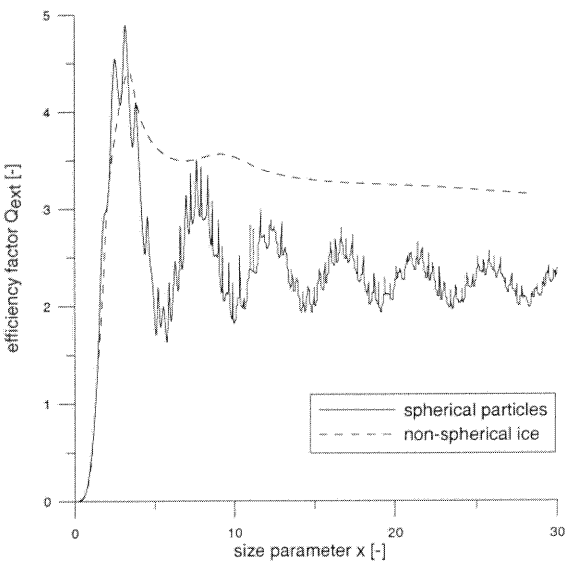

Fig. 3. Efficiency factor for extinction (ice hydrometeors) via dimensionless size parameter.

The attenuation of microwaves decreases significantly with frequency decrement in case very small particles are present in atmosphere. The reduction of attenuation coefficient is evident mostly for icy hydrometeors. The different behavior of attenuation coefficient for both, ice particles and water droplets is referred to Q_{ext}. Namely, the efficiency factor for extinction reacts sensitively on changes of particle refractive index. The important differences between attenuation by liquid water and ice particles can be observed in case the modal radius of gamma distribution function is about 0.5 mm: While τ_H(ices) decreases with ν changing from 100 GHz up to 20 GHz, the τ_H(water) growth up in the same frequency range. As a consequence, the rainfall affects the microwave communication systems more effectively than snowfall when the modal radius of hydrometeors is less than approximately 1 mm. In contrary, water droplets predominately influence communication channel in 10-100 GHz frequency range if $r_m > 1$ mm. Whereas the particles sized from about 0.5 to 1 mm are very representative for atmospheric hydrometeors, the obtained results are quite valuable for microwave communication. Attenuation due to hydrometeors may be a factor of importance especially for microwave systems well above 10 GHz or low-availability systems.

Rain droplets have nearly spherical form, but the atmospheric ices may acquire very curious shape [20]. The radiative transfer in the cloud containing the non-spherical ice particles cannot be simulated using Mie theory. An ideal sphere (larger than incident wavelength) is strongly forward scattering, but irregularities redirect rays from forward peak into other phase angles. We calculated the efficiency factors Q_{ext} for non-spherical ice particles (of aspect ratio 1.4) with use of power Alpha workstation (Fig.3). The behavior of extinction is very similar for non-spherical and spherical ice particles if their radius is less than $\lambda/2$ – it corresponds to the interval $x \in (0,3)$. Non-spherical particles attenuate radiation more effectively at frequencies higher than about 200 GHz.

Although the course of Q_{ext}(sphere) curve fits well the Q_{ext} for non-spherical hydrometeors (Fig. 3), there are still important deviations (Fig. 4).

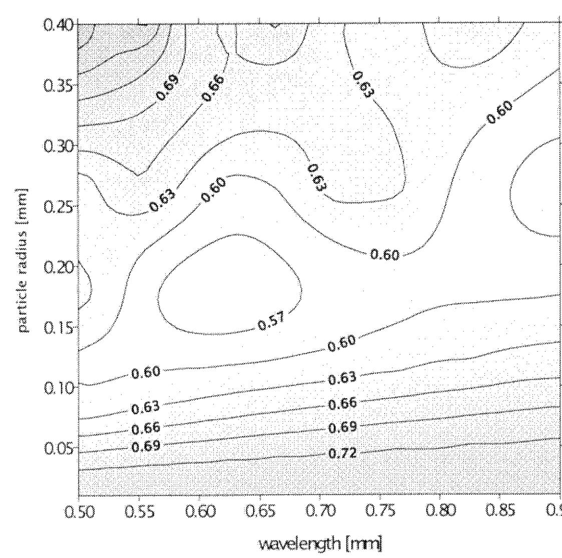

Fig. 4. The function Z(Q) defined as ratio of efficiency factor Q_{ext}(non-sphere) to Q_{ext}(sphere). The computation corresponds to small hydrometeors (aspect ratio 1.4) with refractive index: m=1.59+0i.

We have defined a factor Z(Q) to compare the extinction characteristics of realistically shaped particle with an equivalent sphere. The quantity Z(Q) simply determines the ratio of efficiency factors Q_{ext} for non-spherical and spherical particles. The calculations have shown that Q_{ext}(sphere) is approximately 2 times larger than corresponding value for non-spherical hydrometeors (i.e. $Z \approx 0.5$). This result extends the information given above in the fact that non-spherical particles (in comparison with spherical equivalents) form weakly attenuating media if ν < 150 GHz. The quality of microwave signal transmission through icy cloud is therefore much better as when the communication path is realized through layer containing liquid water droplets (in both cases the particle sizes are assumed to be the same).

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