

PROPAGATION OF SURGE WAVES ON NON-HOMOGENEOUS TRANSMISSION LINES INDUCED BY LIGHTNING STROKE

Z. Benešová, V. Kotlan

UWB Pilsen, Faculty of Electrical Engineering, Department of Theory of Electrical Engineering, Universitní 26,
306 14 Plzeň, Czech Republic
e-mail: bene@kte.zcu.cz, vkotlan@kte.zcu.cz

Summary The paper deals with surge phenomena on non-homogeneous transmission lines. The case of surge phenomena caused by the lightning stroke is considered. The lightning is modeled as the current wave injected by the source placed at any internal point of the line. The propagation of the surge wave induced by the lightning affecting the overhead line and transmitting to the cable is evaluated. Problem is solved numerically in the time domain. The efficiency and correctness of the proposed method is shown on some illustrative examples. According to the suggested algorithms is calculated the distribution of induced current and voltage along the whole transmission line namely near to the place of the connection both lines. The manner of the lines-connection is modeled due to various two-ports.

1. INTRODUCTION

Lightning and switching processes can generate in the power systems fast transient phenomena that result in very high overvoltage. For this reason the lightning surge overvoltage is a dominant factor to determine the insulation level of the whole transmission system. It is very hard to observe the lightning overvoltage experimentally, and thus numerical solution based on EMPT methods has been adopted to investigate it [1], [2] and [3]. If the transmission system consists of two parts - overhead line and cable then the lightning overvoltage on the overhead line can be transmitted on the cable line. The resulting overvoltage at the cable ends has been subject to several investigations [4], [5], while in reality the maximum voltage may occur inside the cable [6]. In [7] authors presents an improved approach where the regular line model is used for obtaining the voltages and currents at the line ends which are next used in the calculation at internal points using an off-line time step. In [8] is presented an analysis of the maximum voltage inside distribution cables from lightning strokes based on analysis by lattice diagram. However, that analysis does not cover the cable attenuation effects found in long transmission cables and does not consider the exact position of maximum voltage. In case of lightning-induced overvoltage in distribution systems, the ultimate overvoltage in the absence of an arrester would be the same as in the transmission system. As a result of these phenomena, a distribution cable connected to an overhead line may see transient waveforms with rise times in the range 100 ns. These short rise time and the large amplitude transients can cause a large voltage across the first few turns of a transformer winding etc. In this paper is suggested new method that enables the complex analysis of propagation of surge waves along the overhead-cable lines.

2. ANALYSIS OF TRANSMISSION LINE IN TIME DOMAIN

2.1 Mathematical model of uniform transmission line

The transient phenomena on transmission line are very fast and so they should be modeled as the network with distributed parameters. On Fig. 1 is depicted general transmission system that consists of three parts: feeding and loading circuits and transmission line of length λ .

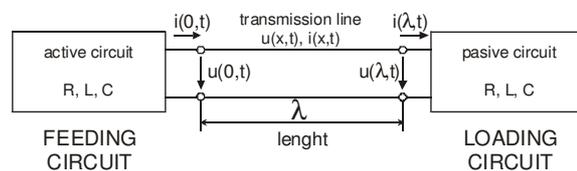


Fig. 1: Schema of the transmission system

In this case voltage and current are time and space varying functions and their distribution along the transmission line can be described by a system of partial differential equations of hyperbolic type. For each element according to Fig. 2 we can write

$$-\frac{\partial u}{\partial x} = Ri + L \frac{\partial i}{\partial t} \quad (1)$$

$$-\frac{\partial i}{\partial x} = Gu + C \frac{\partial u}{\partial t} \quad (2)$$

$$t > 0, \quad x \in \langle 0, \lambda \rangle$$

where R , L , C and G are parameters of the line per unit length. These equations are known as wave or telegraph equations.

To respect the character of the input (feeding circuit) and output (loading circuit) of the line, the system of partial differential equations has to be supplemented by the ordinary differential or algebraic equations

which describe the relationship between current and voltage at the input and output ports – Fig. 1.

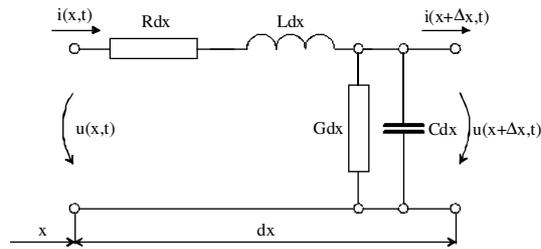


Fig. 2: Schema of one of the elements

For example, if the loading circuit is created by connection of R_2, L_2 we obtain

$$L_2 \frac{di(\lambda, t)}{dt} + R_2 i(\lambda, t) = u(\lambda, t) \quad (3)$$

This type of equations offers the boundary conditions for solution of partial differential equations and guarantee uniqueness of the solution of eq. (1) and (2). The boundary conditions should be supplemented by initial condition, in general form we can write

- **boundary conditions** $t < 0$

$$x=0 \quad F_1 = (u(0, t), i(0, t), t) = 0 \quad (4)$$

$$x=\lambda \quad F_2 = (u(\lambda, t), i(\lambda, t), t) = 0 \quad (5)$$

- **initial conditions:** $t = 0$

$$x \in \langle 0, \lambda \rangle \quad u(x, 0) = 0 \quad (6)$$

$$i(x, 0) = 0$$

2.2 Mathematical model of lightning stroke on transmission line

In a general case, a current source modeling the lightning stroke can be located at any point of the transmission line, it becomes insufficient to respect its value by formulation of the boundary conditions. Therefore, we modify the wave equation for an active element with a current source. The wave equation can be written as

$$\frac{\partial i(x, t)}{\partial x} + G \cdot u(x, t) + C \frac{\partial u(x, t)}{\partial t} = i_0(t) \quad (7)$$

where $i_0(t)$ is a time variable current source that respects the current pulse appearing on the transmission line as the consequence of an atmospheric discharge – the lightning stroke. It can be simulated as a pulse current source [9], the shape of injected current wave can be obtained due to measurement. The current pulse which we used is depicted on Fig.3 and we call it **lightning wave** in spite of the **standard surge wave** $1/50 \mu s$. Both waves differ in the time of duration, in the rate of rise and in the polarity (lightning waves are mostly negative). These differences result in various

magnitudes of injected charge and electric energy supplied to the line. Both shapes of waves were used and the obtained results were compared in [12].

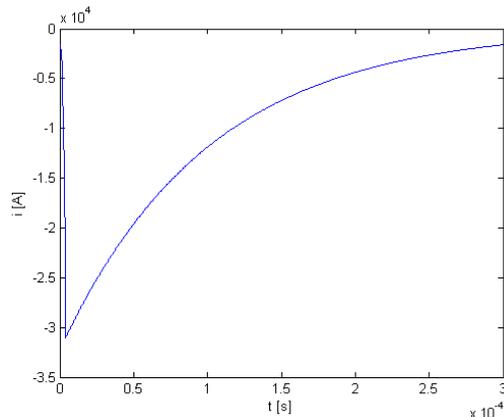


Fig. 3: The shape of lightning wave as a current pulse

2.3 Algorithm for numerical solution

For numerical solution of wave equation (1), (2) and (7) we used the method of finite differences. We supposed that the length of line is divided regularly with the step Δx where $\Delta x = x_{k+1} - x_k$, k denotes the point of line $k = (1, 2, \dots, N)$. Similarly the time step is given by equation $\Delta t = t_\lambda - t_{\lambda-1}$, where λ denotes the time level $\lambda = 1, 2, \dots$

For expression of partial derivations according to time and according to geometric coordinates was used as in thesis [11] and [12] the implicit Wendroff differential formula which reads

$$\frac{\partial v(x, t)}{\partial t} \Big|_{k, l} = \frac{1}{2} \left(\frac{V_k^l - V_k^{l-1}}{\Delta t} + \frac{V_{k+1}^l - V_{k+1}^{l-1}}{\Delta t} \right) \quad (8)$$

$$\frac{\partial v(x, t)}{\partial x} \Big|_{k, l} = \frac{1}{2} \left(\frac{V_{k+1}^l - V_k^l}{\Delta x} + \frac{V_{k+1}^{l-1} - V_k^{l-1}}{\Delta x} \right) \quad (9)$$

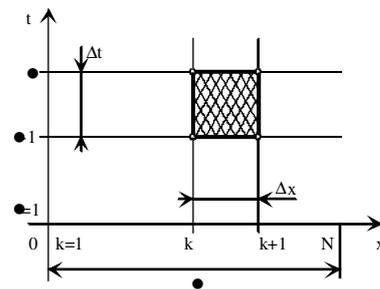


Fig.4 The scheme of Wendroff's formula

We create the differential approximation of standard wave equations (eq. (1) and (2)) for $n - 1$ elements before source and for $m = N - n$ elements behind source. The differential approximation for the active element with the current source has been derived using eq. (1) and (7). For example to express the finite difference schema for eq. (1) at point k and in time level l we obtain

$$\frac{1}{2} \left(\frac{U_{k+1}^l - U_k^l}{\Delta x} + \frac{U_{k+1}^{l-1} - U_k^{l-1}}{\Delta x} \right) + \frac{R}{4} (I_{k+1}^l + I_k^l + I_{k+1}^{l-1} + I_k^{l-1}) + \frac{L}{2} \left(\frac{I_k^l - I_k^{l-1}}{\Delta t} + \frac{I_{k+1}^l - I_{k+1}^{l-1}}{\Delta t} \right) = 0 \quad (10)$$

similarly for eq. (7)

$$\frac{1}{2} \left(\frac{I_{k+1}^l - I_k^l}{\Delta x} + \frac{I_{k+1}^{l-1} - I_k^{l-1}}{\Delta x} \right) + \frac{G}{4} (U_{k+1}^l + U_k^l + U_{k+1}^{l-1} + U_k^{l-1}) + \frac{C}{2} \left(\frac{U_k^l - U_k^{l-1}}{\Delta t} + \frac{U_{k+1}^l - U_{k+1}^{l-1}}{\Delta t} \right) = i_{0k}^l \quad (11)$$

Applying eq. (8) and (9) on eq. (1), (2), (7) and after expressing of boundary value conditions eq.(4) ÷ (6) due to differentiate formulas we obtain a system of 2(N + 1) algebraic equations in matrix form

$$\mathbf{A} \cdot \mathbf{v}^{(\lambda)} = \mathbf{B} \cdot \mathbf{v}^{(\lambda-1)} + \mathbf{D} \quad (12)$$

$\mathbf{v}^{(\lambda)} = [\{u_k\}, \{i_k\}]$ is a matrix of unknown discrete values of voltages and currents in an every space node $k = 1, 2, \dots, N + 1$ of the grid at time l -level and can be evaluated from known values u_k, i_k at λ level, that is at time $t = \Delta t(\lambda - 1)$. Matrix \mathbf{D} respects the values of sources. Matrix equation (12) has been solved due to MATLAB.

3. MATHEMATICAL MODEL OF NON-HOMOGENEOUS LINE

3.1 Model of overhead transmission - cable line interconnection

Now, we will formulate an algorithm for evaluation of voltage and current distribution in system of overhead line interconnected directly or via interconnecting two-port (transformer or arrester etc.) with cable line (Fig. 5).



Fig. 5: Scheme of transmission overhead-cable line and interconnecting two-port

In the case that two lines with various parameters are connected the matrix equation (12) for numerical solution is changed. The matrices \mathbf{A} and \mathbf{B} can be split into submatrices, which respect different parameters of both lines. Except boundary conditions describing relations between voltage and current at input and output of the line it is necessary to complete the equations system with two boundary conditions for the place of interconnection. These

conditions describe relations between input and output parameters of two-port (Fig. 5).

Interconnection via transformer can be modelled in the following manners:

a) T two-port with inductances - (Fig. 6)

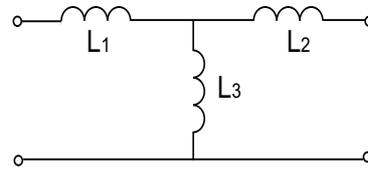


Fig. 6: T two-port with inductances.

We suppose that the transformer is placed at point $N+1$ – see Fig. 5. The equations at input and output ports are

$$\frac{di_{N+1}}{dt} (L_1 + L_3) - L_3 \frac{di_{N+2}}{dt} = u_{N+1} \quad (13a)$$

$$\frac{di_{N+2}}{dt} (L_2 + L_3) - L_3 \frac{di_{N+1}}{dt} = -u_{N+2} \quad (13b)$$

After replacing derivates by differences we obtain

$$\frac{I_k^l - I_k^{l-1}}{\Delta t} (L_1 + L_3) - L_3 \frac{I_{k+1}^l - I_{k+1}^{l-1}}{\Delta t} = \frac{1}{2} (U_k^l + U_k^{l-1}) \quad (14a)$$

$$\frac{I_{k+1}^l - I_{k+1}^{l-1}}{\Delta t} (L_2 + L_3) - L_3 \frac{I_k^l - I_k^{l-1}}{\Delta t} = -\frac{1}{2} (U_{k+1}^l + U_{k+1}^{l-1}) \quad (14b)$$

b) T two-port with capacitance - (Fig. 7, 8).

To observe overvoltage phenomena more convenient and obvious is to take into account the capacitances of transformer, because they play very important role due the great race of rise of voltage wave.

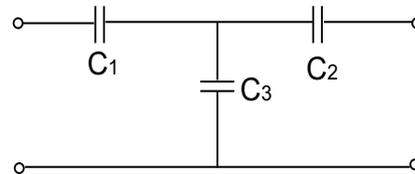


Fig. 7: T two-port with capacitances.

The equation describing relations on input and input port of transformer are

$$u_{N+1} = \frac{1}{C_1} \int i_{N+1} dt + \frac{1}{C_3} \int (i_{N+1} - i_{N+2}) dt \quad (15a)$$

$$u_{N+2} = -\frac{1}{C_2} \int i_{N+2} dt + \frac{1}{C_3} \int (i_{N+1} - i_{N+2}) dt \quad (15b)$$

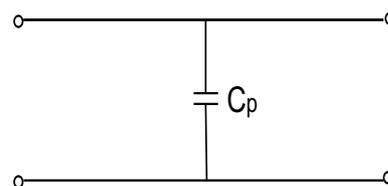


Fig. 8: Two-port with a capacitance.

At first, the very simple model was used – Fig. 8, the equations for input and output of the two-port are in the form

$$\begin{aligned} u_{N+1} &= u_{N+2} \\ i_{N+1} &= i_{N+2} + C \frac{du_{N+1}}{dt} \end{aligned} \quad (16)$$

3.2 Examples and results

The above described algorithm was used for evaluation some interesting examples. The line consisting of two parts was considered:

- the overhead line of length $\lambda_1 = 7,5$ km
parameters: $R_1 = 56,8 \mu\Omega/m$,
 $L_1 = 1,6 \mu H/m$, $C_1 = 6,9$ pF/m,
- the cable of length $\lambda_2 = 2,5$ km
parameters: $R_2 = 0,21$ m Ω/m ,
 $L_2 = 0,54 \mu H/m$, $C_2 = 0,37$ nF/m,

The injected current pulse in the shape of the lightning wave according to Fig.3 was applied at distance $d = 500$ m from the point of interconnection (two-port input). On the following pictures is depicted the current and voltage distribution along the matched line (only reflections at the line interconnection are present) – Fig. 9 and along the unmatched line (input unloaded, output short-circuited) – Fig. 10.

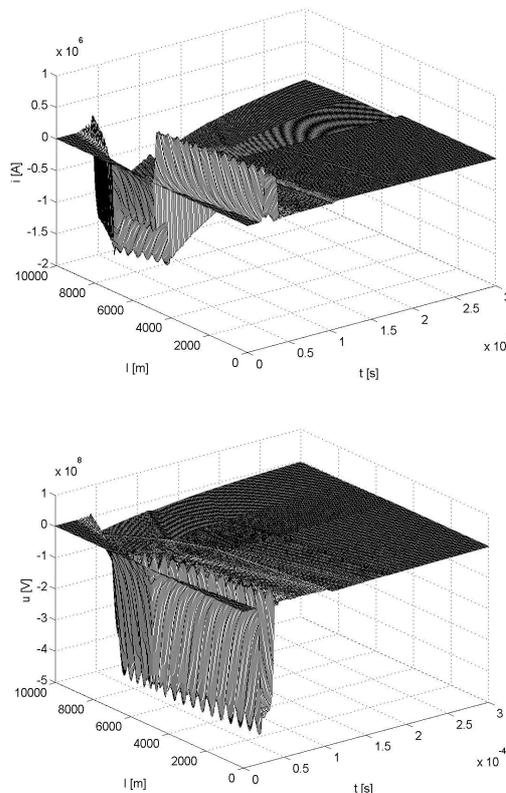


Fig. 9: The distribution of the current and voltage waves on the matched overhead – cable line (direct connection).

The influence of the direct connection manner is evident. The current and voltage waves caused by the lightning can be transmitted from the overhead line to the cable line. The negative influence of direct connection can be limited due to suitable way of interconnection. It is seen from following pictures: on Fig. 11, 12 are depicted results for a transformer modelled by inductances (the matched and unmatched line), on Fig. 13a), b) is seen the time-space distribution of current and voltage along the line for interconnection created by capacitance two-port. It is known that connection overhead-cable line is more dangerous for transmission of current surge wave (due to higher surge impedance of the first part). These phenomena are very good visible on pictures showing the current distribution. The results obtained for an unmatched line show the negative impact of reflections at input and output of the line.

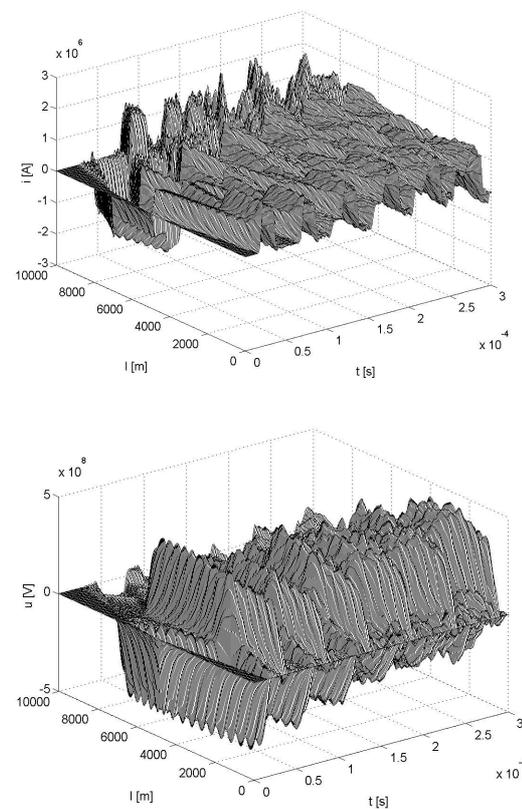


Fig. 10: The distribution of the current and voltage waves on the unmatched overhead – cable line (direct connection)

4. CONCLUSION

In the paper was suggested an algorithm for numerical evaluation of the voltage and current distribution along the non-homogeneous line. In contrast with previous works, the algorithm takes into account the position of the current source simulating the lightning stroke at any internal point of the line. Therefore, the wave equation for an

active element has been derived. Moreover, the proposal algorithm enables to obtain the time-space distribution of voltage and current along the line consisting of two parts and with various ways of interconnection. The work is supplemented with several interesting illustrative examples. Their evaluation indicates a very good correspondence with known phenomena on transmission lines (the matched and unmatched line, the impact of reflected waves, transmission of current and voltage waves via connection two lines with different surge impedances, time of wave propagation etc.). The suggested algorithm is suitable for complex analysis of transient phenomena in many practical applications where the model formed by the network with distributed parameters should be used. For instance, in power energy systems the knowledge of the voltage and current distribution along the line is necessary for the correct adjustment of protections against over-voltage phenomena that appear on transmission lines especially as a consequence of atmospheric discharges.

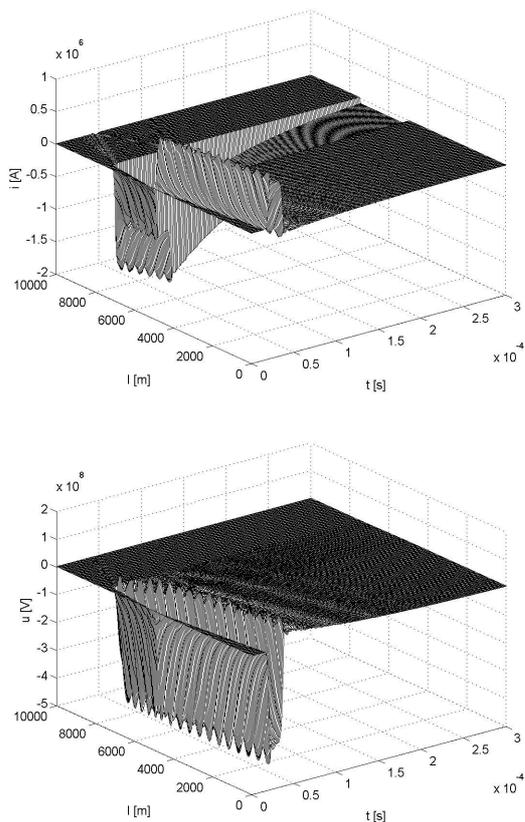


Fig. 11: The distribution of current and voltage waves on matched overhead - cable line connected via transformer (modelled by inductances).

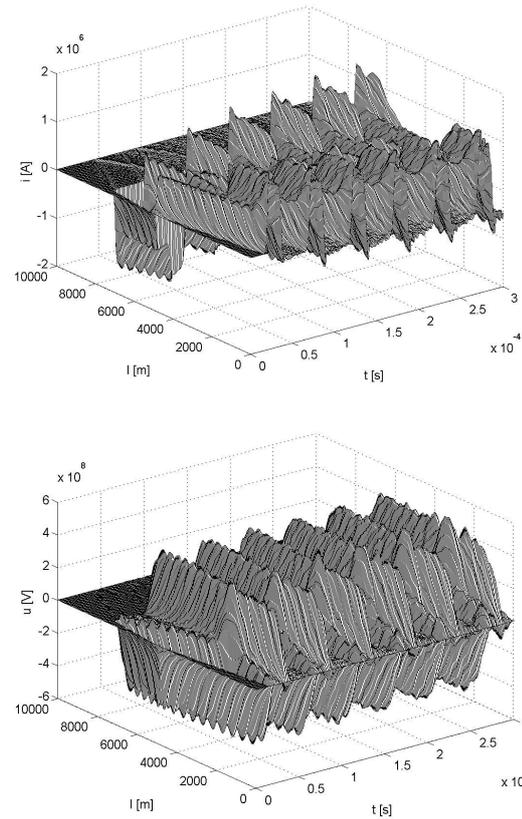


Fig. 12: The distribution of current and voltage waves on unmatched overhead-cable line connected via transformer (modelled by inductances).

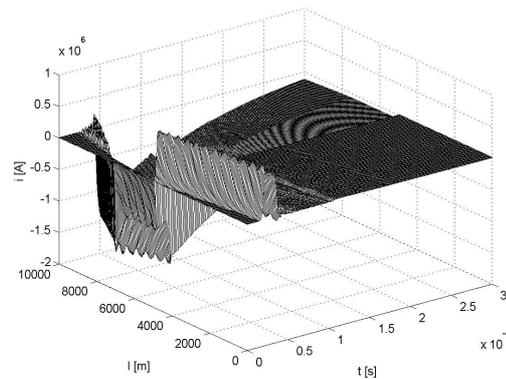


Fig. 13a: The distribution of current wave on matched overhead - cable line with interconnection via transformer (modelled by capacitance $C = C_p = 0,188 \text{ nF}$).

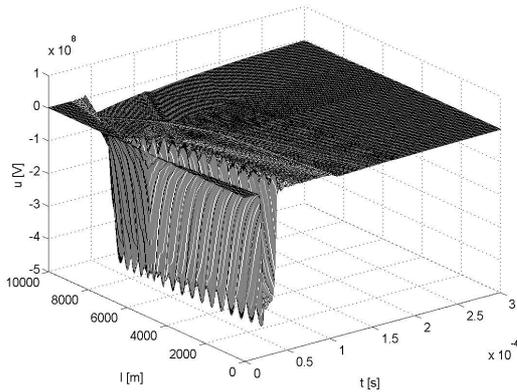


Fig. 13b: The voltage wave distribution along the matched overhead - cable line interconnection via transformer (modelled by capacitance $C = C_p = 0,188$ nF).

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