Use of Chaotic Oscillations for Precoding and Synchronization in OFDM

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Abstract. *This paper proposes a novel linear precoding method for Orthogonal Frequency Division Multiplexing (OFDM) based on the employment of the chaotic waveforms generated by the fourth-order chaotic oscillator and orthonormalized by the Gram-Schmidt process. The proposed linear precoding method is aimed to increase resilience to the multipath propagation issues and reduce the Peak-to-Average Power Ratio (PAPR) of the transmitted signal. Moreover, the chaotic waveform enables novel timing synchronization methods to be implemented in the receiver. The modeling of baseband Linear Precoded OFDM (LP-OFDM) data transmission system with Rayleigh channel has been performed in Simulink environment to validate the proposed method and to compare the performance to the classic precoding methods, such as Walsh-Hadamard Transform (WHT). Experiments have shown that in a high Signal-to-Noise Ratio (SNR) scenario, the employment of the novel precoding scheme allows reducing Bit Error Ratio (BER) by several dB compared to non-precoded OFDM. The proposed precoding method leads to the reduction of PAPR; however, it is not as efficient as classical precoding schemes, such as WHT. Experimental evidence of synchronization of the chaotic oscillators within 50 samples long time interval is presented.*

Keywords

Chaotic synchronization, linear precoding, OFDM, unitary transforms.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most widely used modulation schemes nowadays. Considering spectral efficiency and high accuracy of the equalization, OFDM offers an elegant way to overcome the problem of multi-path propagation and implement high-speed wired and wireless communication systems. However, high spectral efficiency comes at the cost of several challenging issues. Firstly, high Peak-to-Average Power Ratio (PAPR) of the transmitted signal puts very high requirements for communication system power budget and linearity. In communication systems where the transmission medium is nonlinear, for example, Power Line Communication (PLC) and Visible Light Communication (VLC), this problem becomes particularly challenging [1]. The second drawback of OFDM is the very low Signal-to-Noise Ratio (SNR) of some subcarriers due to destructive interference caused by multi-path propagation. A similar situation occurs in the case of narrowband interference caused by other communication systems or hardware imperfections.

One of the approaches that solve both of the mentioned problems simultaneously is the linear transformation of the data symbols before the multicarrier modulation. Linear transformation, which in this case is referred to as LP, allows to spread of information over several subcarriers and, therefore, prevents loss of information due to the low SNR of some subcarriers. Moreover, it is possible to design LP schemes that lead to the reduction of waveform PAPR and other effects, such as spectrum shaping [2]. Linear precoding can be applied not only to Cyclic Prefix OFDM (CP-OFDM) but also to other multicarrier modulations, such as Unique Word OFDM (UW-OFDM) [3], Generalized Frequency Division Multiplexing (GFDM) [4], Wavelet (WOFDM) [5] as well as OFDM-based index modulations [6].

There is a large variety of linear precoding methods described in the literature, depending on the application where they are employed. A fair comparison of different Linear Precoding (LP) matrices for the PAPR reduction is presented in [7]. In the previous decade, much attention has been paid to simple precoding schemes using Walsh-Hadamard Transform (WHT) [8], Discrete Hartley Transform (DHT) [9] and other well-known orthonormal matrices.

The use of Discrete Fourier Transform (DFT) for the precoding of the OFDM, which uses Inverse Discrete Fourier Transform (IDFT) for the modulation, leads to the creation of a single-carrier waveform with minimal PAPR. This feature is widely exploited in many precoders. For example, the authors of [10] propose to combine Gaussian integer sequences and DFT. In publication [11], the authors successfully use Inverse Fast Fourier Transform (IFFT) based precoding to mitigate periodic noise in PLC. In paper [12], authors explore DFT precoding for circular M-QAM constellations. The adaptive frequency domain precoding scheme reported in [13] employs Singular Value Decomposition (SVD) for the diagonalization of the channel matrix and resembles the approach commonly used for the precoding in Multiple-Input Multiple-Output (MIMO) systems. General precoding scheme for OFDM with adjustable PAPR is proposed in [14]. There are special precoders [15] for VLC which allow to overcome problems caused by nonlinearity of light emitting diodes.

Non-periodic broadband signals produced by chaotic circuits have characteristics that make them suitable for scrambling and precoding. There are plenty of examples where chaotic waveforms or chaotic sequences are employed for the LP. In a publication [16], authors propose to use chaotic scrambling to reduce PAPR of the transmitted OFDM signal. In the paper, [17] the authors present chaotic interleaving in conjunction with a neuro-fuzzy system for noise cancellation in OFDM system in PLC. In papers [18] and [19] the researchers present novel methods which employ chaos to increase the immunity of OFDM against nonlinear distortion of Light-Emitting Diode (LED) in VLC communication systems.

Linear precoding using orthogonalized chaotic sequences generated by the Logistic map has been reported by our research group in [20]. Recent publication [21] goes even further. The authors propose to use DFT to diagonalize matrix consisting of cyclically shifted chaotic sequences generated by the Logistic map. The resulting precoding matrix is orthogonal, and it is efficiently employed for linear precoding. Finally, the precoding can also be performed after the multicarrier modulation [22]. In this case, precoding is performed in the time domain and leads to different properties of the transmitted waveform.

OFDM employs IDFT for multicarrier modulation and DFT for multicarrier demodulation. Since DFT operates on vectors of discrete samples, it is necessary to synchronize symbol output in the transmitter and input in the receiver. This fact leads to the necessity to use an additional layer of the synchronization - symbol synchronization [23]. In the classic OFDM systems, this type of synchronization is achieved by correlating repeating parts of OFDM symbols with Cyclic Prefix (CP). The chaotic synchronization [24] is a nonlinear phenomenon that allows the synchronization of two similar chaotic systems by sending one of the state variables to another chaotic system. Chaotic signals are useful for synchronization in communication systems [25] and also allow the construction of multiple access systems [26]. In paper [27] authors provide a very detailed analysis of a novel synchronization algorithm, where the controller uses Fourier series for the uncertainty estimation. Also, authors of another research [28] demonstrate a secure and novel digital communication scheme, which employs chaotic synchronization for the decryption of the information. This paper describes an approach where chaotic sequences are used simultaneously for two purposes. Firstly, chaotic sequences are used for linear precoding of OFDM signal for improving Bit Error Ratio (BER) and reducing PAPR. Some preliminary results of this aspect were previously reported in [20]. Secondly, we demonstrate that the same chaotic waveforms can be used for Non Data Aided (NDA) timing synchronization in the OFDM communication system.

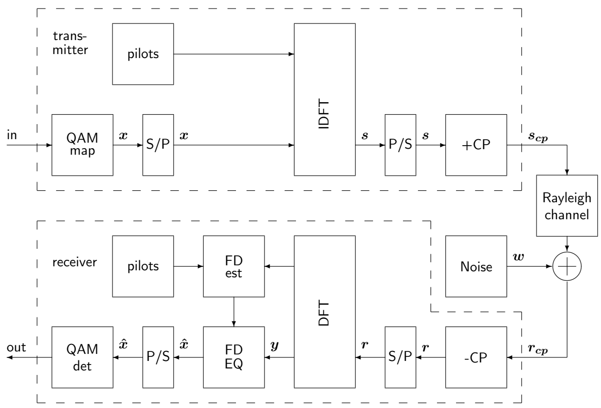

Fig. 1: Conventional OFDM baseband model.

This article is organized as follows: the Sec. 2. provides an overview and mathematical background of OFDM with linear precoding. The Sec. 3. describes and compares the simulation results. In Sec. 3.2. BER performance results are presented, whereas in Sec. 3.3. , PAPR measurements are analyzed. In Sec. 3.4. the problem of chaotic oscillator synchronization is explored. The conclusion section is devoted to the overview of the obtained results.

2. System Model

2.1. Model of Conventional OFDM System

A brief description of the OFDM system is given in the current section. The OFDM system belongs to an Multicarrier (MC) system class. The informational data flow is split into parallel flows with lower data rates and transferred on mutually orthogonal subcarriers. Figure 1 shows a structure of the OFDM system,consisting of two main parts - Quadrature Amplitude Modulation (QAM) mapper and detector as well as OFDM transmitter and receiver. Firstly information bits are mapped by QAM mapper, then converted to parallel flow by serial to parallel (S/P) converter. Payload and pilot signals are transferred on N subcarriers employing IDFT operation:

$$\vec{s} = \mathbf{T}^{-1}\vec{x}, \tag{1}$$

where $\mathbf{T}^{-1}$ is the IDFT matrix. PS converter is used for the OFDM signal transformation into a serial flow. To eliminate Inter-Symbol Interference (ISI) impact on system performance to the beginning of the each OFDM symbol the CP is added before transmission, thus $\vec{s}_{\text{cp}}$ is obtained:

$$\vec{s}_{\text{cp}} = [s_{N-L+1}\ s_{N-L+2}\ \ldots\ s_N\ s_1\ s_2 \ldots s_N], \tag{2}$$

where N is the number of subcarriers and L is the length of the CP. After that, the signal is upconverted to the carrier frequency and sent to the communication channel. The impact of the equivalent baseband Rayleigh channel can be modeled by Finite Impulse Response (FIR) filter with time-varying random complex taps $\vec{h}$ and Additive White Gaussian Noise (AWGN), denoted as $\vec{w}$. Therefore, the received baseband signal is described as follows:

$$\vec{r}_{\text{cp}} = \vec{h} * \vec{s}_{\text{cp}} + \vec{w}, \tag{3}$$

where $*$ denotes circular convolution. At the receiver, the CP of the received symbol $\vec{r}_{\text{cp}}$ is removed:

$$\vec{r} = [r_{\text{cp}\ L+1}\ r_{\text{cp}\ L+2}\ \ldots\ r_{\text{cp}\ L+N}]. \tag{4}$$

Then the symbol is converted into a parallel vector, and for QAM detection transformed into the Frequency Domain (FD) by the DFT operation:

$$\vec{y} = \mathbf{T}\vec{r}, \tag{5}$$

where $\mathbf{T}$ is the DFT matrix while the signal without the CP is $\vec{r}$. A frequency selective channel leads to the linear distortion of the transmitted OFDM signal, resulting in loss of orthogonality between the subcarriers and Inter-Carrier Interference (ICI) and it leads to growth of BER. To improve the performance of the OFDM system, signal equalization in the FD is employed:

$$\hat{x}_k = \frac{r_k}{\hat{H}_k}. \tag{6}$$

The estimate of channel frequency response, vector $\hat{\vec{H}}$, which in case of ideal channel estimation is equal to the DFT of the channel impulse response, is calculated using the reference pilot-tones. If channel characteristic is obtained, the linear distortion, i. e., the ISI caused by the channel, can be eliminated by the linear equalizer. After equalization and parallel to serial conversion, the signal is demodulated.

2.2. Model of Linearly Precoded OFDM System

The Linear Precoded OFDM (LP-OFDM) system is obtained via the insertion of an inverse orthogonal transform before the IDFT in the transmission side and an orthogonal transform after the DFT and the Frequency-Domain Equalizer (FDE) in the receiver. Thus, the information bits are being spread over the whole frequency band of the OFDM symbol before the IDFT operation. Improved frequency diversity of the OFDM signal allows the decreasing impact of frequency selective channel and reduction of the signal PAPR.

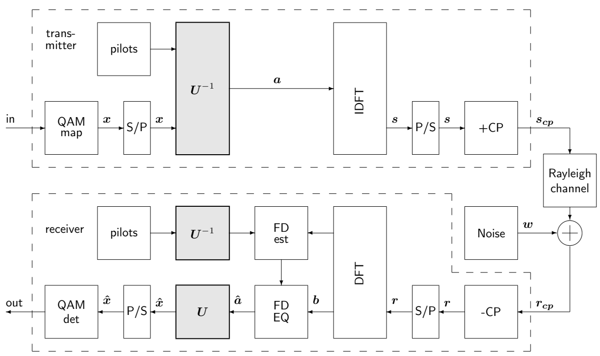

Fig. 2: Linearly precoded OFDM System Model with pilot signal precoding.

Current research is devoted to the study of the novel precoding method, based on Othogonalized Circulant Transform (OCT). The results are compared with the classical approach based on WHT. Figure 2 presents the structure of LP-OFDM system model.

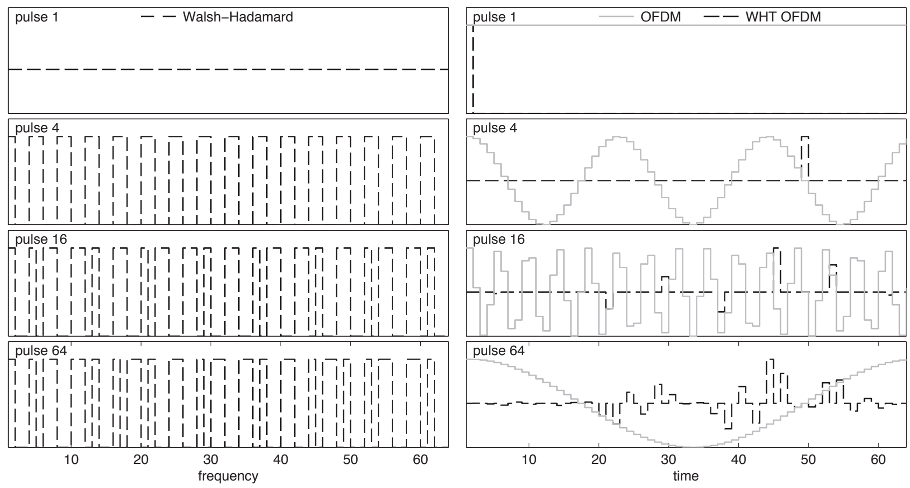

Fig. 3: Pulse shapes of 64 sample WHT. Pulses on the right plot are obtained by applying IDFT to the WHT basis functions, shown on the left plot.

1) Walsh-Hadamard Transform (WHT)

The WHT is one of the orthogonal transforms, which carries out an orthogonal, linear, involutional, symmetric operation on 2^n numbers. The n-th order real matrix of WHT is defined as follows:

$$\mathbf{U}_n = \mathbf{U}_1 \otimes \mathbf{U}_{n-1} = \mathbf{U}_1 \otimes^n, \tag{7}$$

where

$$\mathbf{U}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{8}$$

Examples of FD and Time Domain (TD) pulse shapes, obtained by WHT precoding, are shown in Fig. 3.

2) Orthogonalized Chaotic Transform (OCT)

The OCT, used for the OFDM precoding, is based on the chaos phenomenon and orthonormalization. Such distinct properties of chaotic sequences as non-periodicity and the possibility of synchronization can be efficiently exploited in an MC communication systems. The first step is the generation of a chaotic sequence, and there is a large variety of generation algorithms. Since we plan to use the chaotic sequence for the chaotic synchronization, the chaos generator is based on a modified Chua's circuit. It is a simple electronic circuit exhibiting chaotic behavior, which has been widely studied [29], [30] and [31]. This circuit has been extensively studied by our research group also in the context of Chaos Shift Keying (CSK) [32] and Frequency Modulation CSK (FM-CSK) [33] communication systems.

The dynamics of Chua's circuit can be modeled by means of a set of three nonlinear ordinary differential equations in the variables $p_1(t)$, $p_2(t)$ and $p_3(t)$:

$$\begin{cases} \dfrac{dp_1}{dt} = \alpha(p_2 - p_1 - g(p_1)), \\ \dfrac{dp_2}{dt} = p_1 - p_2 - p_3, \\ \dfrac{dp_3}{dt} = \beta p_2, \end{cases} \tag{9}$$

where $g(p_1)$ is a nonlinear function:

$$g(p_1) = m_1 p_1 + \frac{1}{2}(m_0 - m_1)[|p_1 + 1| - |p_1 - 1|], \tag{10}$$

and α and β are real numbers. By varying α and β parameters, one can observe the period-doubling bifurcation route to chaos. Chua's circuit, for the first time, was described more than 30 years ago, and from that time, many modifications of Chua's circuit have been introduced [34], [35] and [36]. Therefore, to include the possibility of utilizing chaotic sequences for synchronization of the LP-OFDM system, we have chosen one of the modified Chua's circuit versions - a fourth-order chaos oscillator, whose dynamics is described by a set of four nonlinear differential equations in the variables $p_1(t)$, $p_2(t)$, $p_3(t)$ and $p_4(t)$:

$$\begin{cases} \dfrac{dp_1}{dt} = -g(p_1 - p_3) - p_2, \\ \dfrac{dp_2}{dt} = p_1 + \gamma p_2, \\ \dfrac{dp_3}{dt} = \theta(g(p_1 - p_3) - p_4), \\ \dfrac{dp_4}{dt} = \sigma p_3, \end{cases} \tag{11}$$

where $g(p_1 - p_3)$ is a nonlinear function:

$$g(p_1 - p_3) = \begin{cases} c(p_1 - p_3 - d) & \text{if } (p_1 - p_3) \le d \\ 0 & \text{if } (p_1 - p_3) > d \end{cases}, \tag{12}$$

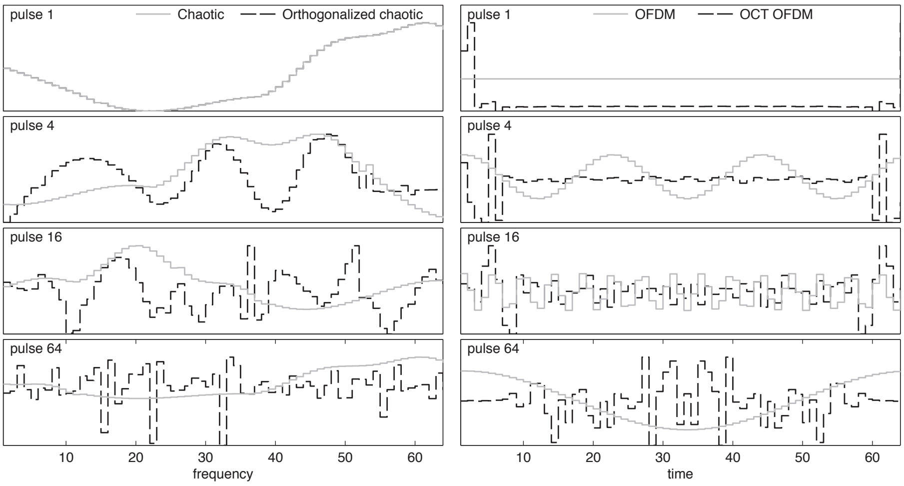

Fig. 4: Pulse shapes, obtained by 64 sample grouping of composite signal from Chua's circuit. Pulses on the right plot are obtained by applying IDFT to the OCT basis functions shown on the left plot.

and γ, θ, σ, c and d are real numbers. In order to generate the chaotic sequence, which could also be used for synchronization, the weighted sum of all state variables with weights $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ (given in Tab. 2) and the nonlinear function is used:

$$R = \left(\lambda_1 \frac{dp_1}{dt} + \lambda_2 \frac{dp_2}{dt} + \lambda_3 \frac{dp_3}{dt} + \lambda_4 \frac{dp_4}{dt}\right) + g(p_1 - p_3) \tag{13}$$

The length of the generated vector $\vec{R}$, which is a discrete-time version of the function R, is restricted to MN^2 elements. This sequence is decimated by a factor M:

$$\tilde{\vec{R}} = [R_1\ R_M\ R_{2M}\ \ldots\ R_{MN}\ R_{M(N+1)}\ \ldots\ R_{MNN}]. \tag{14}$$

The decimated chaotic sequence $\tilde{\mathbf{R}}$ of length N^2 is then reshaped into matrix $\mathbf{V}$ with dimensions $N \times N$. The number of of OFDM subcarriers is equal to the number and the length of the sequences . Thus, chaotic matrix is defined as follows:

$$\mathbf{V} = [\vec{v}_{1x} \quad \vec{v}_{2x} \quad \ldots \quad \vec{v}_{Nx}] = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \\ v_{N+1} & v_{N+2} & \cdots & v_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ v_{(N-1)N+1} & v_{(N-1)N+2} & \cdots & v_{NN} \end{bmatrix}. \tag{15}$$

Despite chaotic sequences having low cross-correlation, they are not entirely orthogonal; therefore, the chaotic matrix $\mathbf{V}$ is being orthonormalized via the Gram-Schmidt process [37]:

$$\vec{z}_1 = \vec{v}_1, \qquad \vec{u}_1 = \frac{\vec{z}_1}{\|\vec{z}_1\|}, \tag{16}$$

$$\vec{z}_2 = \vec{v}_2 - \phi_{\vec{z}_1}(\vec{v}_2), \qquad \vec{u}_2 = \frac{\vec{z}_2}{\|\vec{z}_2\|}, \tag{17}$$

$$\vec{z}_3 = \vec{v}_3 - \phi_{\vec{z}_1}(\vec{v}_3) - \phi_{\vec{z}_2}(\vec{v}_3), \qquad \vec{u}_3 = \frac{\vec{z}_3}{\|\vec{z}_3\|}, \tag{18}$$

$$\vec{z}_4 = \vec{v}_4 - \phi_{\vec{z}_1}(\vec{v}_4) - \phi_{\vec{z}_2}(\vec{v}_4) - \phi_{\vec{z}_3}(\vec{v}_4), \qquad \vec{u}_4 = \frac{\vec{z}_4}{\|\vec{z}_4\|}, \tag{19}$$

$$\vdots \qquad \vdots$$

$$\vec{z}_k = \vec{v}_k - \sum_{j=1}^{k-1} \phi_{\vec{z}_j}(\vec{v}_k), \qquad \vec{u}_k = \frac{\vec{z}_k}{\|\vec{z}_k\|}, \tag{20}$$

where

$$\phi_{\vec{z}}(\vec{v}) = \frac{\langle \vec{v}, \vec{z} \rangle}{\langle \vec{z}, \vec{z} \rangle}\vec{z}, \tag{21}$$

is projection of vector $\vec{v}$ on the vector $\vec{z}$. The results of Gram-Schmidt process are stored in the matrix $\mathbf{U} = [\vec{u}_{1x}\ \vec{u}_{2x}\ \ldots\ \vec{u}_{Nx}]$, which is generated via the orthogonal vector set $\vec{z}_k$ normalization. It is important to remember that the orthogonalization process keep the first row of OCT matrix unchanged.

Examples of pulses before and after Gram-Schmidt orthonormalization for non-decimated chaotic sequences and $1/10$ decimated chaotic sequences are shown in Fig. 4 and Fig. 5, respectively. The values of parameters used for the generation of chaotic precoding sequences are presented in Tab. 1 and Tab. 2.

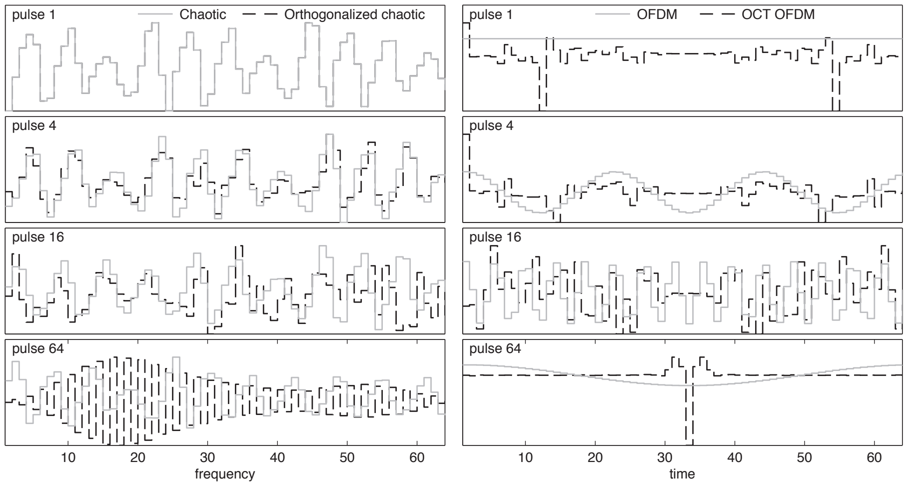

Fig. 5: Pulse shapes, obtained by 1/10 decimation and 64 sample grouping of composite signal from Chua's circuit. Pulses on the right plot are obtained by applying IDFT to the OCT basis functions shown on the left plot.

Orthonormalized chaotic sequences are basis functions of the OCT, and these sequences can be stored in a memory table instead of generating them each time, therefore, reducing the complexity of implementation. On the other hand, the synchronization feature of chaotic sequences opens the possibility of continuously changing basis functions. The computational complexity of the offered OCT precoder is around N^2 additions and N^2 multiplications, since the orthonormalized matrix $\mathbf{U}$ is impossible to factorize into smaller matrices.

Tab. 1: Parameters of chaotic sequence generation for the OCT.

Initial conditions				Time step	Decimation factor	Length
p_1	p_2	p_3	p_4	dt	M	N
0.05	0.06	0.07	0.08	0.1	1	64
0.05	0.06	0.07	0.08	0.1	10	64

Tab. 2: Coefficients of differential equations.

γ	θ	σ	c	d	λ_1	λ_2	λ_3	λ_4
0.5	10	1.5	3	1	-2.63	-0.605	-0.587	0.776

It is worth mentioning that some other orthogonalization processes have the potential to give different results. For example, the orthonormalization processes [38] and [39] proposed by our research group generate a set of orthogonal waveforms from one reference. In this case, the created precoding matrix can be factorized and, therefore, fast precoding algorithms are available.

3) Design of LP-OFDM Model

Figure 2 presents a model of the LP-OFDM system. The inverse orthogonal transform $\mathbf{U}^{-1}$, respectively WHT or OCT, is used to spread the QAM modulated data and pilot signal $\vec{x}$ over all samples of FD signal $\vec{a}$:

$$\vec{a} = \mathbf{U}^{-1}\vec{x}, \tag{22}$$

where $\mathbf{U^{-1}}$ is the inverse orthogonal transform (in the current case the OCT or the WHT). Some examples of basis functions for the WHT and the OCT transform were shown on the left side of Fig. 3, Fig. 4, and Fig. 5. Inverse and direct WHT transforms are equal, as the WHT matrix is Hermitian. The LP-OFDM time domain signal $\vec{s}$ is formed as the precoded informational signal $\vec{a}$ multiplication by the IDFT:

$$\vec{s} = \mathbf{T}^{-1}\vec{a} = \mathbf{T}^{-1}\mathbf{U}^{-1}\vec{x}, \tag{23}$$

where $\mathbf{T}^{-1}$ is the IDFT matrix. The results of applying IDFT to some basis functions (as examples) for the WHT and the OCT transforms were shown on the right sides of Fig. 3, Fig. 4, and Fig. 5. After these operations signal parallel to serial conversion is done and according to (2) the CP is added before each OFDM symbol - $s\vec{c}p$. The signal experiences the same impact (3) of the communication channel as in case of OFDM.

In the receiver firstly the CP is removed according to (4) and then received signal is converted into the parallel vector $\vec{r}$. The next step of the signal processing is the DFT:

$$\vec{b} = \mathbf{T}\vec{r}, \tag{24}$$

where $\mathbf{T}$ is the DFT matrix and $\vec{b}$ is the received precoded informational signal. The channel estimation and equalization in the FD is done by units "FD est" and "FD EQ" in Fig. 2. The estimated symbols $\hat{a}_k$ are obtained by division of the received FD symbols b_k by estimate of channel frequency response $\hat{H}_k$:

$$\hat{a}_k = \frac{b_k}{\hat{H}_k}. \tag{25}$$

Since pilot signals in the transmitter were precoded, for the estimation of $\hat{H}_k$ it is necessary to use precoded versions of the pilot signals.

Finally, the reconstruction of the QAM data samples $\hat{\vec{x}}$ from the spread equalized ones $\hat{\vec{a}}$ is performed by the direct orthogonal transform:

$$\hat{\vec{x}} = \mathbf{U}\hat{\vec{a}}, \tag{26}$$

where $\mathbf{U}$ is the orthogonal transform (respectively the OCT or WHT). And last step in receiver signal processing is binary data detection via the QAM detector.

Spreading of the pilot tones by the precoder may reduce equalization efficiency. To explore this aspect, the model of the LP-OFDM system without pilot signals precoding, shown in Fig. 6, has been constructed as well.

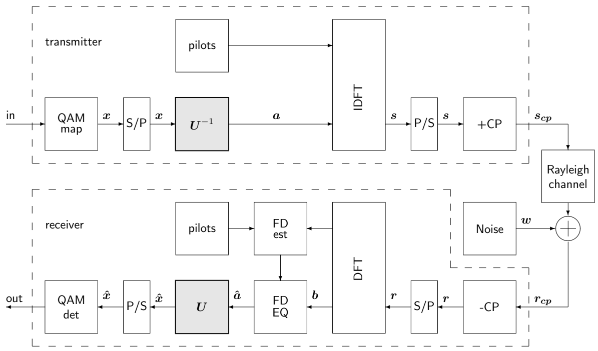

Fig. 6: Linearly precoded OFDM system model without pilot signals precoding.

3. Simulation Results

3.1. Parameters of Simulation

The validity of the proposed precoding scheme has been verified using computer simulations in MATLAB Simulink environment. Two models of baseband LP-OFDM communication systems have been created - with pilot tone precoding, as shown in Fig. 2, and without pilot tone precoding, as shown in Fig. 6. Each model had the possibility to switch among three different precoding matrices: unit matrix (no precoding), WHT matrix, OCT matrix. The transmission was carried out using 24 symbol frames consisting of 20 data symbols and 4 training symbols for block-type FD channel estimation and Zero Forcing (ZF) equalization [40]. The dispersive communication channel was modeled using a model of baseband Rayleigh channel with 4 complex taps changing once per frame and AWGN. Perfect timing and frequency synchronization between OFDM transmitter and receiver was ensured by employing identical and synchronous clock signals. A summary of the simulation setup is given in Tab. 3.

Tab. 3: Parameters of simulated communication systems.

Parameter	Value
Total number of subcarriers	64
Number of data subcarriers	64
Length of the CP	16
Number of data symbols per frame	20
Number of training symbols per frame	4
Total number of symbols per frame	24
Training sequence	ZC [41]
Number of bits transferred	10^8

3.2. Bit Error Ratio Performance

Figure 7 shows BER plots for different precoding schemes and pilot tone insertion methods. From the figure can be seen that if the scheme without pilot tone precoding is used, the linear precoding leads to improvement of the communication system, as BER decreases in both cases using WHT matrix and using OCT matrix, especially in high SNR mode. On the other hand, the linear precoding of pilot tones leads to degradation of the communication system and higher BER compared to non-precoded OFDM. As it can be seen from Fig. 8, the linear precoding causes loss of pilot tone amplitude uniformity, which is the main advantage of Zadoff-Chu sequences [41] used for the channel estimation. Lower amplitudes of some pilot tones lead to degradation of channel estimation accuracy on some frequencies and higher average BER of the communication system.

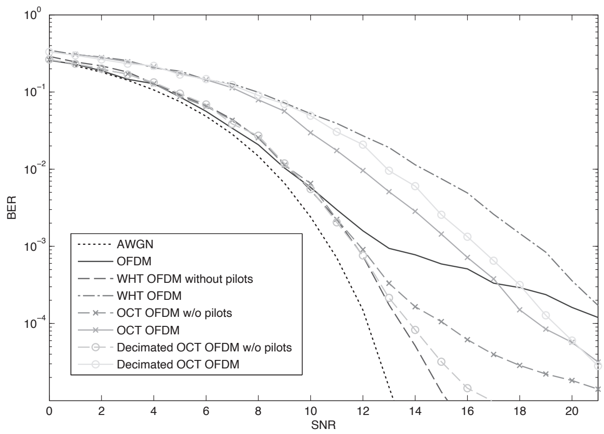

Fig. 7: Performance of the MC communication systems with different modulation schemes.

3.3. Peak-to-Average Power Ratio Measurements

The Complementary Cumulative Distribution Functions (CCDFs) of time-domain signals have been calculated to measure the impact of linear precoding on the PAPR of the transmitted signal. As it can be seen from Fig. 9, all methods of linear precoding lead to the reduction of PAPR of the transmitted waveform. This is especially pronounced in the case of linear precoding using WHT matrix, which leads to a reduction of peak value by approximately 1.5 dB. Whereas OCT reduces peak value and narrows the distribution of the time domain signal just slightly.

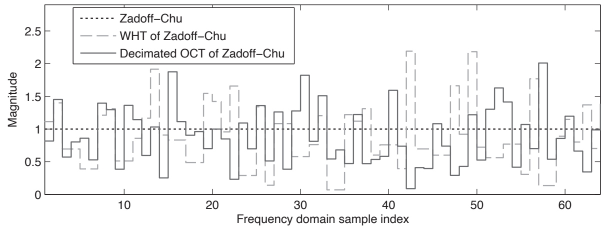

Fig. 8: Magnitude of frequency domain training signals.

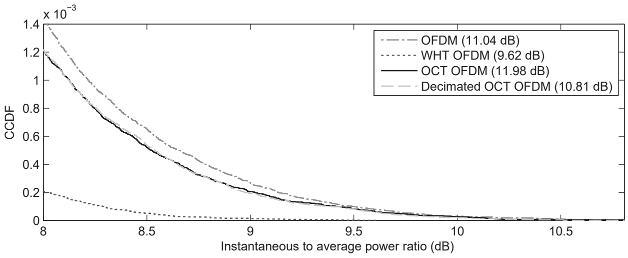

Fig. 9: CCDF tails of the baseband signals in the communication channel.

3.4. Chaotic Synchronization

To check the applicability of the proposed LP sequences for chaotic synchronization, a discrete-time model of chaotic drive system - response system pair has been studied. Chaotic synchronization is well explored for continuous-time systems [28], whereas OFDM is based exclusively on the discrete-time signal processing. This is the main problem of chaotic synchronization in the LP-OFDM system.

We have tested the impact of sampling and decimation on the chaotic synchronization between two continuous-state discrete-time models of modified Chua's circuits (11). In our research, we used the so-called observer-based chaotic synchronization [42] as shown in Fig. 10. The experimental setup is as follows: sampled chaotic sequences (13) are generated off-line by the modified Chua's circuit model. They are used for OCT-based LP (see Sec. 2.2.). This research tested two versions of chaotic sequences: based on non-decimated output from the modified Chua's circuit ($M = 1$) and on decimated ($M = 10$) one. For synchronization, we employ only $2N$ samples of $\tilde{\mathbf{R}}$, i.e., the first two rows of $\mathbf{V}$.

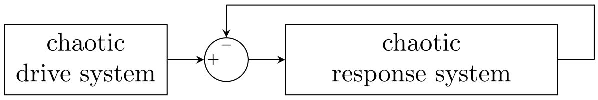

Fig. 10: Observer-based chaotic synchronization.

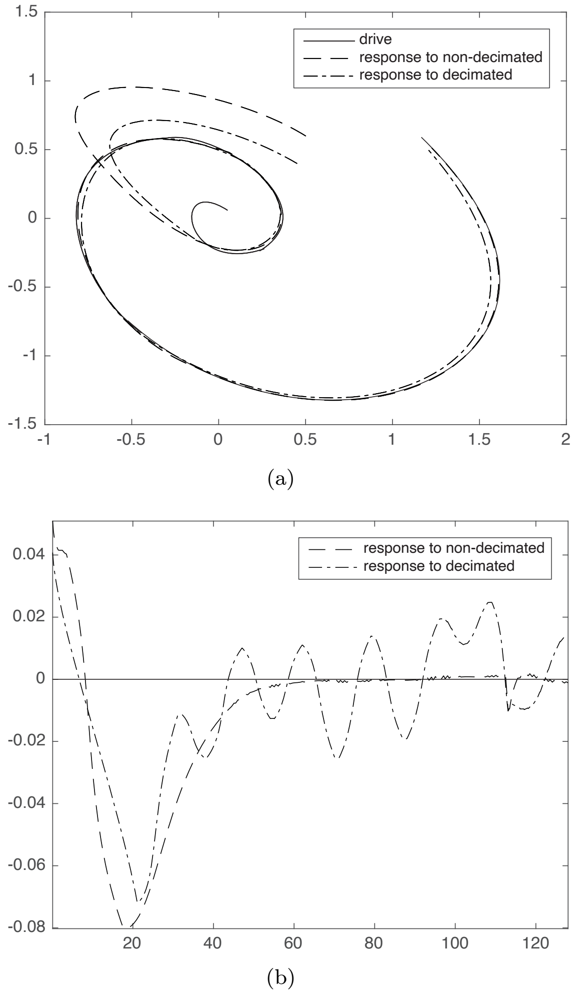

Fig. 11: Phase trajectories of the chaotic systems (a) and synchronization error (b).

In the first experiment, the sampled and recorded output of the drive chaotic system consisting of 128 samples is sent to the response chaotic system model. Phase trajectories of the drive and response systems, initialized at different conditions, show stable synchronization within approximately 50 samples. In the second experiment, we used 10 times decimated (i.e., low-pass filtered and downsampled) output of the drive system. Before sending it to the response system, the sequence is 10 times interpolated. The aim of the experiment is to observe the impact of decimation/interpolation on chaotic synchronization. The experiment shows that the response system, which is started at random initial conditions, can still synchronize, although with reduced accuracy. Phase trajectories of the drive and response chaotic systems, as well as error plots, are shown in Fig. 11.

Those encouraging results show that chaotic synchronization is stable enough to be used for synchronization in the OFDM receiver. Synchronization can be implemented by transmitting of data symbol consisting of one non-zero element, which will lead to the transmission of FD pattern corresponding to the chaotic waveform of the respective row of the OCT matrix. There is at least one row of OCT matrix that is not changed by the Gran-Schmidt orthogonalization process. The received pattern can be used to synchronize the chaotic oscillator in the receiver. After that, the receiver's sample timing and symbol timing clocks can be derived from the synchronized chaotic signal. However, implementing such a synchronization algorithm is worth separate publication and will not be presented here.

4. Conclusion

This paper proposes a novel, chaotic waveforms-based linear precoding method for OFDM. Besides the reduction of communication system error rate and improvement of time domain signal PAPR, the proposed chaotic precoding method offers new means for the synchronization in OFDM receiver.

In the high SNR scenario, when SNR exceeds 10 dB, the proposed linear precoding scheme demonstrates improvement of communication system throughput compared to the non-precoded OFDM case. If we compare the given precoding method with WHT-based precoding, it gives us similar results in terms of BER.

It was found that precoding of training signals leads to a remarkable increase of the BER since the LP destroys a uniformity of pilot tone amplitudes. Therefore, the pilot tones for channel estimation must be excluded from the precoding.

Linear precoding by the selected OCT waveform leads to an insignificant reduction of PAPR. Further minimization of PAPR is possible by the employment of methods that increase frequency domain signal diversity, presented in [3].

Stable synchronization between discrete-time chaotic oscillators can be achieved within 50 samples, providing the possibility of timing synchronization of OFDM receiver. The experiments have shown that chaotic sequences can be decimated before using them as precoding and synchronization sequences. Appropriate interpolation of the sequence must be performed before sending it to the synchronization response system.

The proposed OCT-based precoder increases security aspects of the communications system and, therefore, has potential for secure applications. The use of chaotic synchronization significantly increases the diversity of encryption mechanisms.

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Author Contributions

A.A. developed the concept of linear precoding and orthogonalization of the chaotic sequences, A.L. is coauthor of the idea, she also worked on the mathematical models of the chaotic oscillators and their synchronization, M.T. developed MATLAB/Simulink scripts for the simulations and helped with the typesetting of the manuscript, A.A. contributed to multicarrier communication, equalization, and synchronization in OFDM.

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